The Effects of Increased Competition in a Vertically Separated Railway Market

Markus Lang
Marc Laperrouza
Matthias Finger

Swiss Economics Working Paper 0025
October 2010
The Effects of Increased Competition in a Vertically Separated Railway Market

Abstract

This paper presents a game-theoretic model of a liberalized railway market, in which train operation and ownership of infrastructure are vertically separated. We analyze how the regulatory agency will optimally set the charges that operators have to pay to the infrastructure manager for access to the tracks and how these charges change with increased competition in the railway market. Our analysis shows that an increased number of competitors in the freight and/or passenger segment reduces prices per kilometer and increases total output in train kilometers. The regulatory agency reacts to more competition with a reduction in access charges in the corresponding segment. Consumers benefit through lower prices, while the effect on the operators’ profits is ambiguous and depends on the degree of competition. We further show that social welfare always increases through more competition in the freight and/or passenger segment. Finally, social welfare is higher under two-part tariffs than under one-part tariffs if raising public funds is costly to society.

Keywords: Access charge, optimal pricing, railways, regulation, vertical integration

JEL Classification: D40, L22, L51, L92

*We wish to acknowledge useful comments and suggestions on previous drafts by Helmut Dietl, Martin Grossmann, Matthieu Lapparent, Emile Quinet, Urs Trinkner, seminar participants at EPFL and conference participants at the European Transport Conference 2010 in Glasgow, UK. Financial assistance was provided by a grant of the Swiss National Science Foundation (Grant No. 100014-120503). The authors are solely responsible for the views expressed here and for any remaining errors.
1 Introduction

The introduction of competition in the European railway market lies at the center of the reforms initiated by the European Commission. Competition was expected to play several roles: revitalize the sector, increase efficiency among the railway firms as well as have positive spillover effects on the European economy in general. As a general rule in Europe, one can observe more competition in freight than in the passenger segment. For instance, the incumbent operator SBB Cargo has lost more than 10% market share between 2006 and 2009 for transalpine rail freight passing through Switzerland (SBB 2010). In Romania, private freight firms have captured 25% of the total ton-kilometer, whereas the figure stands at 15% in Poland (Pittman et al. 2007). The situation is not identical in the passenger segment as only very few countries have witnessed the emergence of competition on the tracks (e.g., United Kingdom). Notwithstanding structural reasons this can be explained by an earlier mandated opening of the freight segment to competition.\(^1\) Except for the United Kingdom, which is characterized by an oligopoly of private train operating companies, long-distance passenger services are by-and-large dominated by the incumbent operators (Beckers et al. 2009).

In addition, access to the rail infrastructure is a crucial component of the European railway liberalization process (Gibson et al. 2002; Crozet 2004; ECMT 2005; Nash 2005). For instance, the European Union legislation requires Member States to separate the rail infrastructure from operations and to calculate access charges for the use of the rail infrastructure on a transparent and non-discriminatory basis.\(^2\) The First Railway Package required Member States to separate the management of infrastructure, freight and passenger services into separate divisions with their own profit and loss accounts and balance sheets.\(^3\) While no particular organizational model was required by the EU Directives, one can identify three alternative models of railway restructuring: complete separation, the holding company and the separation of key powers (Nash 2008). Although the exact degree of separation between infrastructure and operations differs across countries, complete separation is the most commonly used restructuring scheme in Europe. It has been adopted by Member States in northern and western Europe.

Access charges to the rail infrastructure should be set in a way that encourages efficient use while avoiding discrimination among similar users (Thompson and Perkins 2006). In practice, one can observe large difference in access charges between freight and passenger transport and across European countries. Member states follow three broad models for

\(^{1}\) Freight was fully opened to competition as of January 1st, 2007. International passenger services are open since January 1st, 2010.


\(^{3}\) The First Package comprised Directives 2001/12, 2001/13 and 2001/14.
infrastructure access charges (OECD 2005): (i) social marginal cost pricing, in which the state covers the difference between total financial costs and revenues, (ii) the full financial cost minus subsidies in which access charges are set to cover the difference between state transfers and the full financial cost and (iii) mark-ups to social marginal costs, which serves both efficiency goals and budgetary pressures. In addition, the structure of access charges can be divided into simple and two-part tariffs. In the former case, prices are set in relation to the usage of the network (e.g., train-kilometer or gross-ton kilometer). In the latter case, operators pay a mixture of fixed and variable prices (Freebairn 1998).

In short, access charges remain an important issue for the European railway policy in its attempt to ensure non-discriminatory access to the existing network. At the same time, they play an important role in determining the competitiveness of new railway lines (Sánchez-Borràs et al. 2010). It is therefore not surprising that access charges have drawn significant interest at the theoretical level (Dodgson 1994; Bassanini and Poulet 2000; Nash 2001; Quinet 2003; Link 2004; Erhan and Robert 2005). While the existing literature has focused largely on cost-allocation methods, empirical studies, and analytical studies of access charges in a vertically integrated market, this paper presents a game-theoretic model of a liberalized railway market in which train operation and ownership of infrastructure is fully vertically separated. In particular, we apply non-cooperative game theory to model the interactions between decision-makers in the railway industry to determine their optimal behavior. Our model incorporates operators, consumers, the regulatory agency and the infrastructure manager. We further differentiate two segments in the railway market: the passenger segment and the freight segment. Moreover, our analysis features a two-stage setup: in the first stage, the regulatory agency sets access charges to maximize social welfare and in the second stage, the operators simultaneously maximize their profits.

The objective of this paper is to analyze how the regulatory agency will optimally set access charges to the infrastructure and how this price-setting behavior changes with increased competition in the railway market. Moreover, we explicitly assess the effect of increased competition on the price per kilometer, the outputs and profits of the operators, consumer surplus, and finally, we assess the welfare implications. The paper is of interest to operators, infrastructure managers, regulators and policy makers in the railway industry because recommendations can be derived on how to optimally set access charges from a social welfare perspective.

The remainder of the paper is structured as follows. Section 2 presents the model framework of a separated railway market and introduces its main actors. In Section 3, we solve the maximization problems of the operators and the regulatory agency. In Section 4, we analyze the effects of more competition. Section 5 extends the model to two-part tariffs.
tariffs and discusses different objective functions of the regulatory agency. Finally, Section 6 discusses the main findings and concludes the paper.

2 A Model of a Vertically Separated Railway Market

We present a simple model of a railway market in which train operation and infrastructure management are fully vertically separated. As noted above, this scenario represents the situation most often encountered in Europe. In the following subsections, we introduce the main actors in the railway market, i.e., operators, consumers, the infrastructure manager, and the regulatory agency.

2.1 Operators

We consider two segments: the freight segment and the passenger segment. In segment $k \in \{f, p\}$ there are $n_k \in \mathbb{N}^+$ symmetric operators active. Following the literature, we model the competition in segment $k$ as Cournot competition (e.g., Baumol 1983; Quinet and Vickerman 2004; Friebel and Gonzalez 2005).

The demand function in segment $k$ is defined as:

$$Q_k = \theta_k - p_k,$$

where $Q_k = \sum_{i=1}^{n_k} q_{ik} \in \mathbb{R}_0^+$ is the total output in train kilometers in segment $k$ and $q_{ik} \in \mathbb{R}_0^+$ is the individual output in train kilometers of operator $i \in \{1, \ldots, n_k\}$ in segment $k$. The parameter $\theta_k \in \mathbb{R}^+$ denotes the market volume, and $p_k \in \mathbb{R}_0^+$ is the price that consumers have to pay for rail services per kilometer in segment $k \in \{f, p\}$. The inverse demand function is thus given by $p_k = \theta_k - \sum_{i=1}^{n_k} q_{ik}$. Note that we abstract from capacity problems on the railway network. Moreover, our model posits that mixed traffic (i.e., both passenger and freight) is allowed on the network.

Operators have to pay a charge to the infrastructure manager for access to the infrastructure (tracks). We assume that the infrastructure manager charges operators and that the regulatory agency sets linear access charges ($a_f, a_p) \in \mathbb{R}_0^+$ per train kilometer in the freight and passenger segments, respectively. Here, our assumption is that the regulatory agency is entrusted with balancing the transport budget and maximizing the overall social welfare.

5If not otherwise stated, the parameter $k$ denotes the segment with $k \in \{f, p\}$. The subscript $f$ stands for the freight segment, while $p$ denotes the passenger segment.

6In Section 5.1, we extend our framework and analyze two-part tariffs which are composed of a variable and fixed part.

7The results are qualitatively unchanged for the case that the infrastructure manager prices access and the charges are then reviewed by the regulatory agency.
Operator $i$ in segment $k$ realizes profits $\pi_{ik}$ according to the following profit function:

$$\pi_{ik} = (p_k - a_k)q_{ik} - c_{ik}(q_{ik}).$$  \hspace{1cm} (2)

The revenues of an operator in segment $k$ are given by the difference between the price $p_k$ charged to its consumers minus the access charge $a_k$ paid to the infrastructure manager, times the output $q_{ik}$ in train kilometers. Furthermore, each operator incurs costs through the operation of its trains (e.g., maintenance and personnel) given by a convex cost function $c_{ik}(q_{ik})$ with $\frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} > 0$ and $\frac{\partial^2 c_{ik}(q_{ik})}{\partial q_{ik}^2} \geq 0$.

### 2.2 Infrastructure Manager

We assume that the infrastructure manager incurs costs through the maintenance of the railroad network according to the following cost function (Kennedy 1997):

$$c_{IM} = F + v_f \left( \sum_{i=1}^{n_f} q_{if} \right) + v_p \left( \sum_{j=1}^{n_p} q_{jp} \right),$$  \hspace{1cm} (3)

where $F \in \mathbb{R}^+$ denotes the fixed network costs, and $v_k(\cdot)$ is a cost function representing the unit-variable part of the infrastructure costs depending on the total output $Q_k = \sum_{i=1}^{n_k} q_{ik}$ in train kilometers of rail services in segment $k \in \{p, f\}$.

We assume a convex cost function $v_k(\cdot)$ with $\frac{\partial v_k(\cdot)}{\partial Q_k} > 0$ and $\frac{\partial^2 v_k(\cdot)}{\partial Q_k^2} \geq 0$.

The profit function $\pi_{IM}$ of the infrastructure manager is then given by:

$$\pi_{IM} = T + \sum_{i=1}^{n_f} a_{fi}q_{if} + \sum_{j=1}^{n_p} a_{pj}q_{jp} - c_{IM},$$  \hspace{1cm} (4)

where $T \in \mathbb{R}^+_0$ denotes total transfers from the government to the infrastructure manager to guarantee that she/he breaks even. As mentioned above, the split of activities among infrastructure managers and operators varies across countries, depending on the type of organizational model. The different degrees of separation affect the responsibilities in terms of investment, timetabling, maintenance and renewal, train control and safety (Nash 2008).

### 2.3 Consumers

Our model assumes that consumers regroup both segments, i.e., individuals in the passenger transport segment and freight rail customers. Consumer surplus $CS_k$ in segment

---

8 The costs of the infrastructure manager can be referring to maintenance and operation costs but they can also encompass renewals or part of the investment needs (CER and EIM 2008).
$k \in \{p, f\}$ is given by the integral of the demand function from the equilibrium price $\hat{p}_k$ to the maximum price $p'_k$ that consumers are willing to pay for rail services in segment $k$:

$$CS_k = \sum_{i=1}^{n_k} \left\{ \int_{\hat{p}_k}^{p'_k} (\theta_k - p_k) \, dp_k \right\}.$$  \hspace{1cm} (5)

### 2.4 Regulatory Agency

The final actor in the model is the regulatory agency. Such regulatory bodies come in different forms and are entrusted with different powers throughout Europe. For instance, in the United Kingdom, the Office of Railway Regulation (ORR) has been operating independently for many years. In France, the railway authority (Autorité de régulation des activités ferroviaires or ARAF) was created at the end of 2009 but has yet to start operations. In some cases the agencies are explicitly entrusted with the supervision of access charges (e.g., ORR). In other cases, their remit is defined much more loosely, such as the supervision of opening to competition.

The regulatory agency sets access charges such that it maximizes social welfare under the constraint that the infrastructure manager realizes non-negative profits. We further assume that the regulator agency has perfect information about all variables in the model.\(^9\) Governments are concerned with ensuring that the infrastructure manager breaks even. Because the latter is usually not in a position to do so, the regulatory agency has to find a financial equilibrium by mixing partial cost recovery (charged to the passenger and freight operators) and governmental transfers to the infrastructure manager. These lump sum transfers $T$ to the infrastructure manager are costly to society because raising public funds is associated with deadweight losses, which are represented in our model by the parameter $\lambda \in \mathbb{R}^+_0$ (c.f. Kennedy 1997; Friebel and Gonzalez 2005). The sources of funds for the transfers to the infrastructure manager often come from different budgets. For instance, in Switzerland, the operation and maintenance costs are part of one budget, while the construction of new lines is taken care of by a different one.

Social welfare is given by the sum of aggregate operator profits and consumer surpluses in the freight and passenger segments minus governmental transfers to the infrastructure manager:

$$W = \Pi_p + \Pi_f + CS_f + CS_p - (1 + \lambda)T,$$

where $\Pi_k = \sum_{i=1}^{n_k} \pi_{ik}$ denotes aggregate profits of the operators in segment $k \in \{p, f\}$.

\(^9\)See Pedersen (1994) for an analysis with private information about costs.
3 Equilibrium Analysis

In this section, we solve the problem of the regulatory agency and the operators. The timing of the model features a two-stage structure.

**Stage 1:** The regulatory agency sets access charges to maximize social welfare under the constraint that the infrastructure manager breaks even.

**Stage 2:** Given access charges set by the regulatory agency in Stage 1, the operators in the passenger and freight segment maximize their profits simultaneously. Finally, payoffs are realized.

We solve for the subgame-perfect equilibria in this two-stage game by applying backward induction.

### 3.1 Maximization Problem of the Operators

First, we consider the Stage 2 maximization problem of the operators in segment $k$ given that the regulatory agency has set linear access charges ($a_f, a_p$) in Stage 1. In segment $k$, operator $i$ solves the following maximization problem:

$$
\max_{q_{ik} \geq 0} \{ \pi_{ik} = \left[ (\theta_k - \sum_{i=1}^{n_k} q_{ik} - a_k \right] q_{ik} - c_{ik}(q_{ik}) \}.
$$

For simplicity and to make the model tractable, we assume that the convex operating costs are given by a quadratic cost function and that the operators in segment $k$ have an equal cost structure, i.e., $c_{ik}(q_{ik}) = \frac{c_k}{2} q_{ik}^2 \forall i \in \{1, \ldots, n_k\}$.

The first-order conditions are then computed as:

$$
\frac{\partial \pi_{ik}}{\partial q_{ik}} = (\theta_k - a_k) - \sum_{j=1, j \neq i}^{n_k} q_{jk} - q_{ik}(2 + c_k) = 0,
$$

yielding the reaction function of operator $i$ as:

$$
R_{ik}(q_{jk}) = \frac{(\theta_k - a_k) - \sum_{j=1, j \neq i}^{n_k} q_{jk}}{2 + c_k}.
$$

The output by operator $i$ decreases with a higher parameter $c_k$ for their own operating costs and higher access charges $a_k$. Similarly, the output also decreases with a higher aggregate output $\sum_{j=1, j \neq i}^{n_k} q_{jk}$ by the other competitors.

Solving the system of reaction functions (8) leads to Lemma 1.

---

10Our results do not change qualitatively if we assume that marginal operating costs are constant (linear cost function). Moreover, we analyze asymmetric operators below, i.e., operators that differ with respect to their operating costs.

11It can easily be verified that the second-order conditions for a maximum are satisfied.
Lemma 1

Given an access charge of $a_k \in \mathbb{R}_0^+$ set by the regulatory agency in the first stage, Stage 2 equilibrium prices and outputs of operator $i \in \{1, \ldots, n_k\}$ in segment $k \in \{f, p\}$ yield:

\[
\hat{p}_k = \frac{n_k a_k + \theta_k (1 + c_k)}{n_k + 1 + c_k} \quad \text{and} \quad \hat{q}_{ik} = \frac{\theta_k - a_k}{n_k + 1 + c_k} \equiv \hat{q}_k. \tag{9}
\]

**Proof.** It is straightforward to derive $\hat{q}_{ik}$ by solving the system of reaction functions (8). Plugging $\hat{q}_{ik}$ into the demand function (1) yields $\hat{p}_k$.

By substituting (9) in the profit function (2), we compute Stage 2 equilibrium profits of operator $i$ in segment $k$ as:

\[
\hat{\pi}_{ik} = \frac{(\theta_k - a_k)^2 (2 + c_k)}{2(n_k + 1 + c_k)^2}. \tag{10}
\]

Due to the symmetry of the operators, each operator in segment $k$ chooses the same output $\hat{q}_{ik}$ in train kilometers. To guarantee that each operator has a non-negative equilibrium output, we assume that $\theta_k \geq a_k$. The lemma further shows that higher access charges $a_k$ in segment $k$ are carried over to the consumers in the form of higher prices $\hat{p}_k$ for train services in segment $k$. The operators increase prices for the consumers less than the access charge increases, i.e., $\partial \hat{p}_k / \partial a_k < 1$: an increase in the access charge of one-unit translates into an increase of consumer prices of less than one. However, with more competition, the increase in prices through a one-unit increase in access charges augments and, in the limit, would converge to one.

Moreover, each operator lowers its output $\hat{q}_{ik}$ in train kilometers in response to higher access charges. Finally, individual profits $\hat{\pi}_{ik}$ of the operators and thus also aggregate profits $\hat{\pi}_k$ in segment $k$, decrease with higher access charges (albeit with a decreasing rate). The reason is that the decrease in costs through a lower output cannot compensate for lower revenues through a lower markup $\hat{p}_k - a_k$.

The consumer surplus in segment $k$ is computed from (5) as:

\[
\hat{CS}_k = \sum_{i=1}^{n_k} \left\{ \left[ \theta_k p_k - \frac{1}{2} \frac{\theta_k}{\hat{p}_k} \right] \hat{q}_{ik} \right\} = \frac{n_k^3}{2} \left( \frac{\theta_k - a_k}{n_k + 1 + c_k} \right)^2. \tag{11}
\]

We derive that the consumer surplus decreases with higher access charges (albeit with a decreasing rate) because prices $\hat{p}_k$ per kilometer increase.

**Asymmetric operators:**

Now, we assume that operators in segment $k$ differ with respect to their operating costs, i.e., $c_{ik} \neq c_{jk}$ with $i, j \in \{1, 2\}$ and $i \neq j$. That is, we relax the assumption regarding the symmetric cost structure of the operators and analyze its effect in a duopoly.
setting \((n_f = n_p = 2)\).

The Stage 2 equilibrium prices and outputs of operator \(i\) in segment \(k\) can be computed from (8) by setting \(c_k = c_{ik}\):

\[
\hat{p}_k = \frac{2a_k(1 + c_{ik} + c_{jk}) + \theta_k (1 + c_{ik})(1 + c_{jk})}{3 + 2(c_{ik} + c_{jk}) + c_{ik}c_{jk}} \quad \text{and} \quad \hat{q}_{ik} = \frac{(1 + c_{jk})(\theta_k - a_k)}{3 + 2(c_{ik} + c_{jk}) + c_{ik}c_{jk}}.
\]

Equilibrium profits of operator \(i\) amount to:

\[
\hat{\pi}_{ik} = \frac{(2 + c_{ik})(1 + c_{jk})^2(\theta_k - a_k)^2}{2(3 + 2(c_{ik} + c_{jk}) + c_{ik}c_{jk})^2},
\]

with \(i, j \in \{1, 2\}\) and \(i \neq j\). It is intuitive that the operator with higher operating costs has a lower market share in equilibrium. Due to its higher marginal costs, the high-cost operator will choose a lower output in train kilometers in equilibrium, i.e., \(\hat{q}_{ik} > \hat{q}_{jk} \iff c_{jk} > c_{ik}\). It follows that the high-cost operator also realizes lower profits in equilibrium.

For the subsequent analysis, we turn back to the general case of \(n_k\) competitors and symmetric operating costs \(c_k\) in segment \(k\).

### 3.2 Maximization Problem of the Regulatory Agency

In Stage 1, the regulatory agency maximizes social welfare \(W\) by anticipating the optimal behavior of the operators in Stage 2. For the sake of simplicity, we assume that the unit-variable costs for the infrastructure manager are given by \(v(\cdot) = \sum_{i=1}^{n_k} vq_{ik}\). That is, the infrastructure manager incurs linear costs per train kilometer, which are equal for freight and passenger trains.\(^{12}\) The maximization problem of the regulatory agency is then given by (c.f. Laffont and Tirole 1994; Armstrong 1996):

\[
\max_{(a_f, a_p) \geq 0} \{ W = \Pi_p + \Pi_f + CS_f + CS_p - (1 + \lambda)T \} \quad \text{subject to}
\]

\[
(i) \; \pi_{IM} = T + (a_f - v) \sum_{i=1}^{n_f} q_{if} + (a_p - v) \sum_{j=1}^{n_p} q_{jp} - F \geq 0 \quad \text{and} \quad (ii) \; T \geq 0.
\]

Constraint (i) is the break-even condition for the infrastructure manager, while constraint (ii) imposes that governmental transfers have to be non-negative. The solution to the maximization problem is derived in the following lemma:

\(^{12}\)Our results do not change qualitatively if we utilize a strictly convex cost function for the infrastructure manager.
Lemma 2

In Stage 1, the regulatory agency will set access charges in segment $k \in \{f, p\}$ as:

$$a_k^* = \frac{(n_k + 1 + c_k)(v(1 + \lambda) + \lambda \theta_k) - \theta_k(1 + n_k(n_k - 1))}{n_k(2 - n_k) + 2\lambda(n_k + 1 + c_k) + c_k}.$$  (12)

Proof. See Appendix A.1. ■

Lemma 2 shows that the regulatory agency will set access charges according to (12). Notice that the break-even condition for the infrastructure manager is satisfied with equality because increasing governmental transfers above the break-even level is costly to society. We further derive that access charges $a_k^*$ increase with higher costs $\lambda$ for raising public funds: to finance the higher costs for the governmental transfers to the infrastructure manager, the regulatory agency sets higher access charges. Similarly, access charges also increase with higher costs $c_k$ for the operators and higher costs $v$ for the infrastructure manager.

Suppose that the passenger and the freight segments have an equal number of competitors and the same market volume, i.e., $n_f = n_p$ and $\theta_f = \theta_p$. In this scenario, equilibrium prices $p_k^*$ and access charges $a_k^*$ are higher in the segment that is characterized by higher operating costs of its operators, while the opposite holds true regarding total equilibrium outputs $Q_k^*$. Formally, $(p_\mu^* > p_\nu^*, a_\mu^* > a_\nu^* \text{ and } Q_\mu^* < Q_\nu^*) \iff c_\mu > c_\nu \text{ for } \mu, \nu \in \{f, p\}, \mu \neq \nu$.

By substituting (12) in equation (9), we compute Stage 1 equilibrium outputs and prices as:

$$q_{ik}^* = \frac{(\theta_k - v)(1 + \lambda)}{n_k(2 - n_k) + 2\lambda(n_k + 1 + c_k) + c_k} \equiv q_k^* \text{ and } p_k^* = \theta_k - n_kq_k^*.$$  (13)

In the next section, we analyze the effects of an increased number of competitors in the freight and/or passenger segment.

4 The Effects of Increased Competition

As noted, the European Commission pushed for the introduction of competition in the railway sector. Although it initially faced strong resistance from Member States, the railway markets are evolving towards increasing competition in both the passenger and freight segments. This transformation is nonetheless still in its initial stage in most Member States, and most railway stakeholders, including the government, will have to adjust to the new landscape and its implications. The separation of infrastructure management from operations, coupled with the arrival of new entrants, changes the economics of the sector by splitting the financial burden of operating a railway network. Our paper makes
a contribution towards this new allocation.

We start by analyzing the effect of increased competition on the access charges set by the regulatory agency:

**Proposition 1 (Access Charges)**

The regulatory agency reacts to an increased number of competitors \( n_k \) in segment \( k \in \{f, p\} \) with a reduction of the access charges \( a^*_k \) in the corresponding sector.

**Proof.** The proof is straightforward by computing the partial derivatives of \( a^*_k \) with respect to \( n_k \) and by noting that \( n_k \geq 1 \).

To observe the intuition behind the result of Proposition 1, recall that the break-even condition for the infrastructure manager is satisfied with equality, that is, \( T^* = F + (v - a^*_f)Q^*_f + (v - a^*_p)Q^*_p \) with \( Q^*_k = \sum_{i=1}^{n_k} q^*_{ik} \). It follows that higher access charges help to reduce governmental transfers to the infrastructure manager, but higher access charges in segment \( k \) also decrease profits of the operators and the consumer surplus in this segment. A higher number of competitors in segment \( k \) increases the positive effect of higher access charges on social welfare through lower governmental transfers \( T^* \), but at the same time, the negative effect through lower operator profits and consumer surplus \((\pi^*_k + CS^*_k)\) increases as well.\(^{13}\) If access charges are relatively high, then the negative effect of increased competition on social welfare dominates the positive effect. Thus, to balance both effects in equilibrium, the regulatory agency must set lower access charges if the number of competitors increases.\(^{14}\)

Furthermore, note that there exists a threshold number \( n^v_k \) of competitors above which access charges \( a^*_k \) are below marginal infrastructure costs \( v \); that is, \( a^*_k < v \iff n_k > n^v_k. \)\(^{15}\)

Having access charges below marginal cost poses a significant budgetary (and political problem) because one of the major drawbacks of marginal cost pricing (short-run or long-run) stems from the fact that railways are experiencing economies of scope, densities of scale and hence that marginal cost pricing does not fully cover costs (Crozet 2004).

Next, we analyze the effect of a higher number of competitors on prices and outputs of the operators:

**Proposition 2 (Prices and Outputs)**

More competition in segment \( k \) reduces the price \( p^*_k \) per kilometer and increases total output \( Q^*_k \) in train kilometers. The effect on individual output \( q^*_{ik} \) of operator \( i \) is positive if the number of competitors in segment \( k \) is sufficiently large with \( n_k > n^v_k \equiv 1 + \lambda. \)

\(^{13}\)Formally, the cross derivatives are given by \( \partial(\partial\pi^*_k/\partial a_k + \partial CS^*_k/\partial a_k)/\partial n_k < 0 \) and \( \partial(\partial T^*/\partial a_k)/\partial n_k < 0. \) Recall that lower transfers have a positive effect on social welfare.

\(^{14}\)To abstract from the possibility that access charges are negative, we assume that the number of competitors is not too large (see the proof of Lemma 2 for more details). For an analysis in which operators are subsidized, see Else (1985).

\(^{15}\)Note that the threshold number is given by \( n^v_k \equiv 1/2 \left( 1 + \lambda + (\lambda(\lambda + 6 + 4c_k) - 3)^{1/2} \right). \)
Proof. See Appendix A.2. ■

The proposition shows that if a segment is characterized by a relatively low number of competitors, an additional competitor induces the incumbent operators to decrease their individual outputs in train kilometers, while the opposite holds true if competition in the segment is relatively high. The intuition is as follows: from the first-order conditions (7), we deduce that marginal revenue \( (\theta_k - a_k) \) of an additional competitor in segment \( k \) increases because access charges decrease. Note that access charges decrease with an increasing rate with a higher number of competitors, i.e., \( \partial^2 a_k^* / \partial n_k^2 < 0 \). On the other hand, marginal cost \( q_k(n_k + 1 + c_k) \) increases linearly with a higher number of competitors. Thus, if competition is high (low) in segment \( k \), then marginal revenue increases more (less) than marginal cost and operator \( i \) reacts with a higher (lower) output in train kilometers. Even though the effect on individual output by operator \( i \) is ambiguous, total output in segment \( k \) will always increase because the higher number of competitors compensates for a possible decrease in individual output. It follows that due to higher total outputs, the equilibrium price per kilometer in segment \( k \) decreases.

By deriving comparative statics of operator profits with respect to the number of competitors, we can establish Proposition 3.

Proposition 3 (Operator Profits)

(i) Individual profits \( \pi_{ik}^* = (1 + c_k/2) (q_{ik}^*)^2 \) of operator \( i \) in segment \( k \) initially decrease with a higher number of competitors until the (global) minimum is reached for \( n_k = n_k' \). By increasing the number of competitors above \( n_k' \), individual profits start to increase.

(ii) Total operator profits \( \Pi_k^* \) in segment \( k \) are (locally) maximized for \( n_k = n_k \) and (locally) minimized for \( n_k = \bar{n}_k \) with \( n_k < \bar{n}_k \).

Proof. See Appendix A.3. ■

Regarding part (i) of the proposition, it is easy to show that the difference between prices and access charges \( p_k^* - a_k^* \) increases (decreases) with a higher number of competitors if \( n_k > n_k' \) (\( n_k < n_k' \)). Thus, if \( n_k > n_k' \), then revenues increase more than costs such that profits increase, while if \( n_k < n_k' \), then revenues decrease more than costs such that profits decrease.\(^{17} \) If raising public funds is not associated with a deadweight loss, i.e., \( \lambda = 0 \), then individual profits of the operators will always increase (note that in this case \( n_k' = 1 \)). We illustrate the result of part (ii) in Figure 1, which depicts total operator profits \( \Pi_k^* \) in segment \( k \) as a function of \( n_k \).

The figure shows that total operator profits in segment \( k \) increase with a higher number of competitors until a (local) maximum is reached for \( n_k \) competitors. By increasing the

\(^{16}\)Due to the symmetry \( q_{ik} = q_k \forall i \in \{1, ..., n_k\} \) and thus the first-order conditions are reduced to \( (\theta_k - a_k) = (n_k + 1 + c_k) \).

\(^{17}\)Recall that the individual output of operator \( i \) in sector \( k \) increases if \( n_k > n_k' \).
number of competitors above $\pi_k$, total operator profits start to decrease until a (local) minimum is reached for $\pi_k$. By further increasing the number of competitors above $n_k$, total operator profits start to increase until the maximum feasible number of competitors is reached. By noting that the partial derivative of total operator profits $\Pi^*_k = n_k \pi^*_ik$ with respect to $n_k$ is given by $\partial \Pi^*_k/\partial n_k = \pi^*_ik + n_k (\partial \pi^*_ik/\partial n_k)$, the intuition is as follows: for $n_k < n_k$ individual profits $\pi^*_ik$ are relatively high, but they decrease with a higher number of competitors.\textsuperscript{18} In this interval, profits of an additional operator compensate for the lower individual profits of the incumbent operators, such that total profits increase. This holds true until the local maximum is reached for $n_k = n_k$. For $n_k \in (n_k, \pi_k)$, the decrease in $\pi^*_ik$ outweighs additional profits of the entrant operator, such that total profits decrease until the local minimum is attained for $n_k = \pi_k$. For $n_k > \pi_k$, the increase in profits of an additional operator compensates for the loss in the incumbents profits, and total profits increase. If individual profits increase, i.e., $n_k > n'_k$, then it is clear that total profits also increase: thus, it always holds that $\pi_k < n'_k$.

In a next step, we analyze how a higher number of competitors affects the consumer surplus and governmental transfers. Proposition 5 summarizes the findings.

**Proposition 4 (Governmental Transfer and Consumer Surplus)**

(i) Governmental transfers to the infrastructure manager initially decrease through a higher number of competitors in segment $k$ until the (global) minimum is reached for $n = n^T_k$. Increasing the number of competitors above this level, increases governmental transfers.\textsuperscript{18} Remember that individual profits $\pi^*_ik$ decrease with $n_k$ for $n_k < n'_k$, albeit with a decreasing rate.
(ii) The consumer surplus $CS^*_k = n_k^2/2(q_{ik}^*)^2$ in segment $k$ always increases through a higher number of competitors.

**Proof.** See Appendix A.4.

Part (i) of the proposition shows that there exists an optimal number of competitors $n_T^k$, such that governmental transfers to the infrastructure manager are minimized, i.e., $n_T^k = \arg\min_{n_k} T$. To observe the intuition behind this result, note that $\partial T^*/\partial n_k = (v - a_k^*)(\partial Q_k^*/\partial n_k) - (\partial a_k^*/\partial n_k) Q_k^*$. We know that $\partial Q_k^*/\partial n_k > 0$ and $\partial a_k^*/\partial n_k < 0$. Moreover, access charges $a_k^*$ are below marginal infrastructure costs $v$ if the number of competitors is sufficiently large with $n_k > n_k^v$. In this case, governmental transfers increase through more competition because total output $Q_k^*$ increases, and with it, the deficit of the infrastructure manager for which the government has to compensate for. It is therefore clear that access charges must be above marginal infrastructure costs to meet in a situation in which a higher number of competitors can induce a decrease in governmental transfers. That is, if $n_k < n_k^v$ and thus $a_k^* < v$, then it is possible that increased competition reduces transfers. In such a scenario, a higher number of competitors induces a decrease in $(v - a_k^*)$ but, at the same time, an increase in total output $Q_k^*$. The second (positive) effect dominates the first (negative) effect, such that governmental transfers decrease until both effects balance each other out for $n_k = n_T^k$.

Part (ii) of the proposition shows that consumers in segment $k$ benefit from a higher number of competitors because the price per kilometer decreases and thus consumers are better off.

Finally, we determine the welfare effect of a higher number of competitors:

**Proposition 5 (Social Welfare)**

Social welfare always increases through increased competition in the passenger segment and/or freight segment.

**Proof.** See Appendix A.5.

The proposition shows that the society benefits from increased competition in one or both segments through higher social welfare. To observe the intuition behind this result, remember that social welfare is given by the sum of aggregate consumer surpluses and operator profits minus governmental transfers to the infrastructure manager. From Propositions 3 and 4, we know that consumers always benefit from more competition through lower access charges, while the effect on operator profits and governmental transfers is ambiguous.

Suppose that $n_k > n_T^k$.\(^{19}\) If the number of competitors in segment $k$ is relatively low with $n_k < n_k^v$, then it is clear that social welfare increases through a higher num-

\(^{19}\)Note that $n_k^v$ is always lower than $n_T^k$, while it depends on $c_k$ and $\lambda$ whether or not $n_k^v \leq n_T^k$. We provide the intuition for the case that $n'_k > n_T^k$. The intuition for the case $n'_k < n_T^k$ is similar.
ber of competitors because consumer surplus and total operator profits increase, while governmental transfers decrease. If competition is moderately high in segment $k$, i.e., $n_k \in (n_k, n_k^T)$, then the positive effect (following an increase in $n_k$) from higher consumer surplus and lower governmental transfers compensates for the negative effect of lower profits such that social welfare increases. In the case that $n_k \in (n_k^T, n_k)$, the negative effect through higher governmental transfers and lower profits is compensated for by the positive effects of higher consumer surplus.\(^{20}\) If $n_k > n_k$, then higher profits and consumer surplus outweigh higher governmental transfers yielding an increase in welfare.

5 Model Extensions

5.1 Two-Part Tariffs

In this section, we extend our initial model by analyzing a situation in which the regulatory agency sets two-part tariffs. That is, in addition to the linear access charge $a_k$, the regulatory agency imposes a lump sum fee $T_{ik}$ for operator $i$ in segment $k$. Two-part tariffs are found in Great Britain, Italy, France, Bulgaria, Hungary, Lithuania and Romania. As can be expected, one can find variations in the charging mechanisms, driven by the level of sophistication desired. For instance, in France, a fixed access charge applies to all traffic in the same way. It is supplemented by a train path reservation fee (per path-kilometer reserved) and a variable charge per train-km). Further charges are levied on the passenger operations (e.g., stops at stations) or freight operations (e.g., by speed of train).

The profit function of operator $i$ in segment $k$ is then given by:

$$\pi_{ik} = (p_k - a_k)q_{ik} - \frac{c_k}{2}q_{ik}^2 - T_{ik}.$$  \(14\)

The lump sum fee $T_{ik}$ goes directly to the infrastructure manager to help him/her to break even, such that the profit function of the infrastructure manager yields:

$$\pi_{IM} = T + \sum_{i=1}^{n_f} (T_{if} + a_fq_{if}) + \sum_{j=1}^{n_p} (T_{jp} + a_pq_{jp}) - c_{IM},$$

where the costs $c_{IM}$ of the infrastructure manager are given by (3).

The maximization problem of the operators in Stage 2 in segment $k$, given that the regulatory agency has set linear access charges $a_k$ in Stage 1, is similar to above. Thus, we obtain the same Stage 2 equilibrium prices and outputs (9), whereas the profits of operator $i$ in segment $k$ are now given by $\pi^*_{ik} = \frac{(\theta_k - a_k)^2(n_k + c_k + 1)}{(n_k + c_k + 1)^2} - T_{ik}$.

\(^{20}\)Note that consumer surplus increases with an increasing rate in $n_k$. 
Similar to above, the regulatory agency maximizes social welfare in Stage 1 by anticipating the behavior of the operators in Stage 2. The maximization problem of the regulatory agency becomes:

$$\max_{(a_f, a_p) \geq 0} \{ \Pi_f + \Pi_p + CS_f + CS_p - (1 + \lambda)T \} \text{ subject to}$$

(i) $$\pi_{IM} = T + \sum_{i=1}^{n_f} \{ T_{if} + (a_f - v)q_{if} \} + \sum_{j=1}^{n_p} \{ T_{jp} + (a_p - v)q_{jp} \} - F \geq 0,$$

(ii) $$\pi_{ik} = (p_k - a_k)q_{ik} - \frac{c_k}{2}q_{ik}^2 - T_{ik} \geq 0 \text{ and (iii) } T, T_{ik} \geq 0.$$

Again, the break-even condition (i) for the infrastructure manager will be satisfied with equality. As opposed to the case with single tariffs, the infrastructure manager receives a lump sum fee $$T_{ik}$$ from operator $$i$$ in segment $$k$$ in addition to governmental transfers $$T$$. The constraint (ii) is a break-even condition on profits of operator $$i$$ in segment $$k$$. The constraints (iii) impose that governmental transfers and lump sum fees have to be non-negative. Because the regulatory agency has no incentives to leave rents to the operators, it will set the lump sum fees $$(T_{i,f}, T_{i,p})$$, such that operator $$i$$ in segment $$k$$ realizes zero profits, i.e., $$T_{ik} = (p_k - a_k)q_{ik} - \frac{1}{2}c_kq_{ik}^2$$. Substituting this last equality in constraint (i) and recalling that this constraint will be binding with equality, the maximization problem can be rewritten as:

$$\max_{(a_f, a_p) \geq 0} \{ CS_f + CS_p - (1 + \lambda) \left[ F + Q_f (v - p_{i,f} + c_f q_{if}) + Q_p (v - p_{ip} + c_p q_{ip}) \right] \}.$$ (15)

By solving the system of first-order conditions derived from the profit-maximization problem, we can show that the regulatory agency will set access charges and the lump sum fee in segment $$k$$ according to:\footnote{The derivation of the optimal access charges is analogous to Lemma 2. A formal proof is available from the corresponding author upon request.}

$$a^{**}_k = \frac{v(1 + \lambda)(n_k + 1 + c_k) - \theta_k(1 + \lambda + n_k^2 - n_k(1 + \lambda))}{n_k(2 - n_k) + \lambda(2n_k + c_k) + c_k} \quad \text{and} \quad T^{**}_{ik} = \frac{(2 + c_k)(\theta_k - v)^2(1 + \lambda)^2}{2 \left[ n_k(2 - n_k) + \lambda(2n_k + c_k) + c_k \right]^2}.$$ with $$k \in \{f, p\}$$. In addition to the linear access charges $$a^{**}_k$$, the regulatory agency demands a lump sum fee $$T^{**}_{ik}$$ from the operators. From the maximization problem (15) we know that this lump sum fee $$T^{**}_{ik}$$ is set such that operators realize zero profits.

We omit the comparative statics because they are similar to the scenario with linear access charges analyzed above. Comparison of the scenario under linear access charges with the one under two-part tariffs leads to Proposition 6.
Proposition 6 (Two-Part Tariffs)
If raising public funds is costly ($\lambda > 0$), access charges in the scenario with two-part tariffs are always lower than in the scenario with single tariffs, yielding a higher level of social welfare under two-part tariffs. If raising public funds is not costly ($\lambda = 0$) access charges and social welfare coincide in both scenarios.

Proof. See Appendix A.6. ■

If raising public funds is costly, the regulatory agency can set lower access charges under two-part tariffs than under single tariffs because the operators contribute to subsidize the infrastructure manager with their lump sum fees. Due to the lower access charges, the infrastructure manager realizes lower revenues, but the lump sum fees paid by the operators always compensate for the lower access charges. As a result, costly governmental transfers to the infrastructure manager can be reduced. The consumers benefit through lower prices, but the operators are worse off because all of their rent is extracted to subsidize the infrastructure manager. It follows that social welfare increases because higher consumer surplus and lower governmental transfers outweigh the lower operator profits. Thus, two-part tariffs enable the regulatory agency to shift the variable component of the access charge to the fixed component, contributing to reduce costly governmental transfers. From a social point of view, it is preferable that the operators subsidize the infrastructure manager through their lump sum fees instead of the government, if raising public funds is costly. If, however, raising public funds is not costly to society, it does not matter from a welfare perspective who subsidizes the infrastructure manager: the operators or the government. In this case, access charges and social welfare do not differ between both scenarios.

5.2 Different Objective Functions of the Regulatory Agency

In this section, we analyze the effect of integrating profits of only certain operators in the objective function of the regulatory agency. For this purpose, we consider a scenario in which there is only one monopolistic operator in the passenger segment and duopoly competition in the freight segment.

We choose this setup because this resembles the situation in many EU countries. In the freight segment, a substantial level of entry has occurred since 2000. While new entrants initially failed to capture large market shares (Steer Davis & Gleave 2005), this is now changing as freight is undergoing a certain level of concentration through mergers and acquisitions (Bozicnik 2009). For instance, there is now fierce competition on the North-South corridor through Switzerland between SBB Cargo and DB Schenker. As noted above, the situation is rather different in the long-distance passenger segment, where incumbent operators tend to dominate the market (Beckers et al. 2009).
The timing is similar to the general case. Setting $n_f = 1$ and $n_p = 2$, we compute Stage 2 equilibrium prices and outputs with the help of Lemma 1 as:

$$\hat{p}_p = \frac{a_p + \theta_p (c_p + 1)}{2 + c_p} \quad \text{and} \quad \hat{q}_p = \frac{\theta_p - a_p}{2 + c_p} \quad \text{(passenger segment)}$$

$$\hat{p}_f = \frac{2a_f + \theta_f (c_f + 1)}{3 + c_f} \equiv \hat{p}_f \quad \text{and} \quad \hat{q}_f = \frac{\theta_f - a_f}{3 + c_f} \equiv \hat{q}_f \quad \text{(freight segment)}$$

It is clear that prices are higher and total output is lower in the passenger segment with only one monopolistic operator than in the case of more than one competitor (c.f. Lemma 1).

We further assume that the regulatory agency either includes profits $\pi_p = (p_p - a_p)q_p - 1/2c_p q^2_p$ of the monopolistic operator (Regime A) or it does not include them (Regime B) in its objective function $GA$.

The maximization problem of the regulatory agency in Stage 1 can thus be written as:

$$\max_{(a_f,a_p) \geq 0} \left\{ GA = \beta \cdot \pi_p + CS_f + CS_p - (1 + \lambda)T \right\} \text{ subject to } \begin{align*}
(i) \quad \pi_{IM} &= T + (a_p - v)q_p + 2(a_f - v)q_f - F \geq 0, \\
(ii) \quad \pi_{i,f}, \pi_p \geq 0 \text{ and } (iii) \ T \geq 0.
\end{align*}$$

where $\beta = 1$ characterizes the case where the regulatory agency includes profits (Regime A), and $\beta = 0$ is the case where it does not include profits (Regime B) in its objective function. Nevertheless, social welfare $W$ includes profits of all operators and is given by

$$W = \pi_p + \pi_{1f} + \pi_{2f} + CS_f + CS_p - (1 + \lambda)T.$$

Comparison of Regimes A and B yields the following results.

**Proposition 7**

(i) Access charges in the passenger segment are higher in Regime B than in Regime A.

(ii) Governmental transfers are higher in Regime A than in Regime B.

(iii) Social welfare is higher in Regime A than in Regime B.

**Proof.** See Appendix A.7. ■

The proposition shows that the regulatory agency sets lower access charges for the monopolistic operator in the passenger segment if its profits are included in the objective function of the regulatory agency (see Part (i)). It is not surprising that the regulatory privileges the monopolistic operator by lowering the access charges for this operator. Moreover, note that the price-setting behavior of the regulatory agency in the freight segment is not affected by the introduction of profits in the passenger segment.

Furthermore, lower access charges in the passenger segment induce lower prices per kilometer in this segment, yielding a higher surplus for consumers of passenger services.
At the same time, the infrastructure manager profits will decrease as a consequence of lower access charges. To finance the infrastructure manager’s higher deficit, the regulatory agency must raise public funds in Regime A (see Part (ii)). Nevertheless, social welfare is higher compared to Regime B because higher governmental transfers are compensated for by a higher consumer surplus in the passenger segment and higher profits of the monopolistic operator (see Part (iii)).

6 Conclusion

In this paper, we develop a game-theoretic model of a liberalized railway market, in which train operation and ownership of infrastructure are fully vertically separated. With our framework, we are able to derive the equilibria for the operators, consumers, the regulatory agency and the infrastructure manager. In particular, our analysis shows that an increased number of competitors in the freight and/or passenger segment reduces the price per kilometer and increases total output in train kilometers. The effect on individual output per operator is positive if the number of competitors in each segment is sufficiently large. Moreover, the prices per kilometer are higher in the segment that is characterized by higher operating costs of its operators, while the opposite holds true regarding total output in train kilometers (under the assumption that both segments have equal market size and the same number of competitors). The operator with higher operating costs has a lower market share in equilibrium. Due to its higher marginal costs, the high-cost operator will choose a lower output in train kilometers in equilibrium. It follows that the high-cost operators realize lower profits in equilibrium.

The regulatory agency reacts to more competition with a reduction in access charges in the corresponding segment. Consumers benefit through lower prices, while the effect on the operator profits is ambiguous and depends on the degree of competition. Individual profits of each operator decrease through a higher number of competitors if competition is not yet very severe. Otherwise, individual profits increase through more competition. Governmental transfers to the infrastructure manager initially decrease through a higher number of competitors until a minimum is reached for an intermediate level of competition. Increasing the number of competitors above this level, increases governmental transfers. We further show that social welfare always increases through more competition in the freight and/or passenger segment.

Moreover, we analyze a scenario in which the regulatory agency sets two-part tariffs: the operators have to pay a lump sum fee in addition to linear access charges per kilometer. We find that access charges under two-part tariffs are lower than under single tariffs, if raising public funds is costly to society because operators subsidize the infrastructure manager with their lump sum fees. Consumers benefit from lower prices, and governmen-
tal transfers can be reduced. Two-part tariffs thus are an effective instrument to extract rents from the operators without harming the consumers. As a result, the level of social welfare is higher under two-part tariffs than under single tariffs. If, however, raising public funds is not costly, access charges and social welfare coincide in both scenarios.

Finally, we discuss the effects of integrating profits of only certain operators in the objective function of the regulatory agency. For this purpose, we consider a scenario with one monopolistic operator in the passenger segment and duopoly competition in the freight segment. We choose this setup because this resembles the situation in many EU countries. By comparing the scenario in which the regulatory agency does not integrate the profits of the passenger operator into the objective function (Regime A) with the scenario in which the regulatory agency includes profits of the passenger operator (Regime B), we derive that access charges for the passenger segment are higher in Regime A than in Regime B, while governmental transfers are higher in Regime B than in Regime A. Our analysis further shows that social welfare is always higher in Regime B than in Regime A.

Our model remains simple and limited. In reality, the pricing mechanisms devised by the various Member States are much more complex. For instance, in the United Kingdom, the Office of Railway Regulation (ORR) has put in place a very sophisticated pricing system. Despite its limitations, our study can be seen as a first step to analyze the effects of more competition in a vertically separated railways market. We encourage further research in this area. For example, a promising avenue for further research might be the integration of so-called congestion charges into our model framework and the analysis of their effects on operator profits, consumer surplus and social welfare.
A Appendix

A.1 Proof of Lemma 2

The break-even condition for the infrastructure manager will be satisfied with equality in equilibrium because increasing governmental transfers above the break-even level is costly to society. The maximization problem of the regulatory agency can thus be rewritten as:

$$\max_{(a_f, a_p) \geq 0} \{W' = \pi_p + \pi_f + CS_f + CS_p - (1 + \lambda)(F + (v - a_f)Q_f + (v - a_p)Q_p)\}, \quad (17)$$

with \(Q_k = \sum_{i=1}^{n_k} q_{ik}\). The first-order conditions of the maximization problem (17) are derived as:

$$\frac{\partial W'}{\partial a_k} = n_k \left(-\frac{(\theta_k - a_k)^2}{(n_k + 1 + c_k)^3}\right) + n_k \left(\frac{n_k \left(\theta_k - \frac{n_k(a_k - a_k)}{n_k + 1 + c_k}\right) - n_k \theta_k}{n_k + 1 + c_k}\right) - (1 + \lambda) \left(\frac{n_k(2a_k - (\theta_k + v))}{n_k + 1 + c_k}\right) = 0,$$

with \(k \in \{p, f\}\). Note that the second-order conditions for a maximum are satisfied if the number of competitors is sufficiently small with

$$n_k < n'_{k,\text{max}} \equiv 1 + \lambda + \sqrt{1 + c_k + \lambda(4 + 2c_k + \lambda)}.$$

Solving the system of first-order conditions yields:

$$a^*_k = \frac{1}{\varphi} \left[(n_k + 1 + c_k)(v(1 + \lambda) + \lambda\theta_k) - \theta_k(1 + n_k(n_k - 1))\right], \quad (18)$$

with \(\varphi \equiv n_k(2 - n_k) + 2\lambda(n_k + 1 + c_k) + c_k\). To guarantee that the access charges are non-negative, we assume that the number of competitors is sufficiently small with

$$n_k < n''_{k,\text{max}} = \frac{1}{2\theta_k} \left(\varphi + [\varphi^2 + 4\theta_k((1 + c_k)[v + \lambda(\theta_k + v)] - \theta_k)]^{1/2}\right).$$

with \(\varphi \equiv (\theta_k + v)(1 + \lambda)\). Thus, we assume that \(n_k < n_{k,\text{max}} \equiv \min\{n'_{k,\text{max}}, n''_{k,\text{max}}\}\) to guarantee that the second-order conditions and the non-negative condition are satisfied.

A.2 Proof of Proposition 2

Let \(\varphi \equiv n_k(2 - n_k) + 2\lambda(n_k + 1 + c_k) + c_k\). To prove that more competition in segment \(k\) reduces the price \(p^*_k\) per kilometer and increases total output \(Q^*_k\) in train kilometers, we
derive partial derivatives with respect to $n_k$ as:

$$\frac{\partial p_k^*}{\partial n_k} = -\frac{1}{\varphi^2} (\theta_k - v)(1 + \lambda)(c_k + n_k^2 + 2\lambda(1 + c_k)) < 0,$$

$$\frac{\partial Q_k^*}{\partial n_k} = \frac{1}{\varphi^2} (\theta_k - v)(1 + \lambda)(c_k + n_k^2 + 2\lambda(1 + c_k)) > 0.$$ 

Furthermore, we compute:

$$\frac{\partial q_{ik}^*}{\partial n_k} = 2\frac{\partial q_{ik}^*}{\partial n_k} = 2\frac{\varphi^2}{\varphi^2} (\theta_k - v)(1 + \lambda)(n_k - (1 + \lambda))$$

and derive that $\frac{\partial q_{ik}^*}{\partial n_k} > 0 \iff n_k > n'_k = 1 + \lambda$ and $\frac{\partial q_{ik}^*}{\partial n_k} < 0 \iff n_k < n'_k$. Thus, the effect on individual output $q_{ik}^*$ of operator $i$ is positive (negative) if the number of competitors in segment $k$ is sufficiently large (small) with $n_k > n'_k$ ($n_k < n'_k$).

### A.3 Proof of Proposition 3

Part (i): To prove the claim, we substitute equilibrium access charges $a_k^*$ in the operator’s profit function (10) and derive that $\pi_{ik}^* = (1 + c_k/2)(q_{ik}^*)^2$, where $q_{ik}^*$ are the Stage 1 equilibrium outputs (13) of operator $i$ in segment $k$. From Proposition 2, we know that $\frac{\partial q_{ik}^*}{\partial n_k} > 0 \iff n_k > n'_k = 1 + \lambda$. Thus, $\frac{\partial q_{ik}^*}{\partial n_k} < 0 \iff n_k < n'_k$ and $\frac{\partial q_{ik}^*}{\partial n_k} > 0 \iff n_k > n'_k$.

We deduce that individual profits $\pi_{ik}^*$ of operator $i$ in segment $k$ initially decrease with a higher number of competitors, until the minimum is reached for $n_k = n'_k$. By increasing the number of competitors above $n'_k$, individual profits start to increase.

Part (ii): To prove the claim, we compute the partial derivative of total profits $\Pi_k^* = n_k\pi_{ik}^*$ with respect to $n_k$ as:

$$\frac{\partial \Pi_k^*}{\partial n_k} = \pi_{ik}^* + n_k\frac{\partial \pi_{ik}^*}{\partial n_k} = \frac{2 + c_k}{2\varphi^3} (\theta_k - v)^2(1 + \lambda)^2 [n_k(3n_k - 2) + 2\lambda(1 - n_k) + c_k(1 + 2\lambda)]$$

We derive:

$$\frac{\partial \Pi_k^*}{\partial n_k} < 0 \iff n_k \in (n_k, \bar{n}_k) \text{ and } \frac{\partial \Pi_k^*}{\partial n_k} > 0 \text{ for } n_k < n_k \text{ or } n_k > \bar{n}_k \text{ with }$$

$$(n_k, \bar{n}_k) = \left(\frac{1}{3} \left(1 + \lambda - \sqrt{\tau}\right), \frac{1}{3} \left(1 + \lambda + \sqrt{\tau}\right)\right) \text{ and } \tau = 1 + \lambda(\lambda - 4) - 3c_k(1 + 2\lambda)$$

Note that if $\lambda < \lambda' \equiv 2 + 3c_k + \sqrt{3(1 + c_k(5 + 3c_k)}$ then $\tau < 0$ and thus $\frac{\partial \Pi_k^*}{\partial n_k} > 0$ for all feasible $n_k$. 

22
A.4 Proof of Proposition 4

Part (i): To prove the claim, we have to show that \( \frac{\partial T^*}{\partial n_k} < 0 \Leftrightarrow n_k < n_k^T \) and \( \frac{\partial T^*}{\partial n_k} > 0 \Leftrightarrow n_k > n_k^T \). Remember that \( T^* = F + (v - a_j)Q^*_j + (v - a_p^*)Q^*_p \). We derive:

\[
\frac{\partial T^*}{\partial n_k} = (v - a^*_j) \frac{\partial Q^*_k}{\partial n_k} > 0 \text{ or } < 0 \frac{\partial a^*_j}{\partial n_k} \frac{Q^*_k}{\partial n_k} > 0.
\]

We define \( z(n_k) := \frac{\partial T^*}{\partial n_k} \) and note that \( z(n_k) \) is a continuous function in \( n_k \). From the discussion of Proposition 2, we know that \( a^*_k \geq v \Leftrightarrow n_k \leq n_k^v \). Thus, \( z(n_k) > 0 \). It follows that \( n_k < n_k^v \) is a necessary condition for \( z(n_k) = 0 \). We compute:

\[
z(0) = \frac{(\theta_k - v)^2(1 + \lambda)(1 - \lambda)(1 + c_k)}{[c_k + 2\lambda(1 + c_k)]^2} < 0 \Leftrightarrow \lambda > \lambda' \equiv \frac{1}{1 + c_k}.
\]

(a) Suppose that \( \lambda > \lambda' \). According to the intermediate value theorem, there exists a number of competitors \( n_k^T < n_k^v \), such that \( z(n_k^T) = 0 \). This proves the claim because \( T^* \) is a convex function in \( n_k \).

(b) Suppose that \( \lambda < \lambda' \). In this case, it holds that \( z(0) > 0 \). It follows that there does not exist a number of competitors \( n_k^T \in (0, n_k^v) \), such that \( z(n_k^T) = 0 \). Thus, \( z(n_k) > 0 \) for all feasible \( n_k \). This completes the proof of part (i).

Part (ii): To prove the claim, we substitute equilibrium access charges \( a^*_k \) in consumer surplus (11) and derive that \( CS^*_k = n_k^2/2 (q^*_k)^2 \). We compute the partial derivative of \( CS^*_k \) with respect to \( n_k \) as:

\[
\frac{\partial CS^*_k}{\partial n_k} = \frac{2n_k^2}{\varphi^3} (\theta_k - v)^2 (1 + \lambda)^2 \left[ n_k (2 + n_k) + 2\lambda (3 + n_k) + 3c_k (1 + 2\lambda) \right]
\]

with \( \varphi \equiv n_k (2 - n_k) + 2\lambda (n_k + 1 + c_k) + c_k \). Thus, \( \frac{\partial CS^*_k}{\partial n_k} > 0 \) because \( \varphi \) is always positive for all \( n_k < \min\{n'_{k,max}, n''_{k,max}\} \). This proves the claim that consumer surplus always increases with a higher number of competitors.

A.5 Proof of Proposition 5

Let \( \varphi \equiv n_k (2 - n_k) + 2\lambda (n_k + 1 + c_k) + c_k \). To prove the claim, we substitute equilibrium access charges \( a^*_k \) in the welfare function (6) and derive the partial derivative of social welfare with respect to \( n_k \) as:

\[
\frac{\partial W}{\partial n_k} = \frac{\partial \pi^*_k}{\partial n_k} + \frac{\partial CS^*_k}{\partial n_k} - (1 + \lambda) \frac{\partial T^*}{\partial n_k} = \frac{1}{2\varphi^2} (\theta_k - v)^2 (1 + \lambda)^2 (c_k (1 + 2\lambda) + n_k + 2\lambda) > 0.
\]
A.6 Proof of Proposition 6

To prove that access charges under two-part tariffs $a_k^\ast$ are lower than access charges $a_k^\ast$ under single tariffs if $\lambda > 0$, we compute:

$$a_k^\ast - a_k^\ast = \frac{\lambda(1 + \lambda)(\theta_k - v)(2 + c_k)(n_k + 1 + c_k)}{\varphi \cdot \tau}$$

with $\varphi = n_k(2 - n_k) + 2\lambda(n_k + 1 + c_k) + c_k$ and $\tau = n_k(2 - n_k) + \lambda(2n_k + c_k) + c_k$. It follows that $a_k^\ast > a_k^\ast$ if $\lambda > 0$, while $a_k^\ast = a_k^\ast$ if $\lambda = 0$.

In the next step, we compare social welfare under single tariffs with social welfare under two-part tariffs. From the maximization problems (17) and (15), we know that social welfare under single tariffs is given by:

$$W^* = \Pi_f^* + \Pi_p^* + CS_f^* + CS_p^* - (1 + \lambda) \left[ F + (v - a_f^*)Q_f^* + (v - a_p^*)Q_p^* \right],$$

while social welfare under two-part tariffs yields:

$$W^{**} = CS_f^{**} + CS_p^{**} - (1 + \lambda) \left[ F - (T_f^{**} + T_p^{**}) + (v - a_f^{**})Q_f^{**} + (v - a_p^{**})Q_p^{**} \right]$$

$$= (1 + \lambda)(T_f^{**} + T_p^{**}) + CS_f^{**} + CS_p^{**} - (1 + \lambda) \left[ F + (v - a_f^{**})Q_f^{**} + (v - a_p^{**})Q_p^{**} \right],$$

with $T_k^{**} = \sum_{i=1}^{n_k} T_{ik}^{**}$. Suppose that $\lambda > 0$: because $a_k^\ast > a_k^\ast$, we derive that $CS_k^{**} > CS_k^\ast$, $Q_k^{**} > Q_k^\ast$ and $T_k^{**} > \Pi_k^\ast$. Independent of whether or not $a_f \leq v$, the higher consumer surplus and operators’ lump sum fees under two-part tariffs compensate for the higher value of $F + (v - a_f^{**})Q_f^{**} + (v - a_p^{**})Q_p^{**}$, such that $W^{**} > W^*$ always holds.

Suppose that $\lambda = 0$: because $a_k^\ast = a_k^\ast$, we derive that $CS_k^{**} = CS_k^\ast$, $Q_k^{**} = Q_k^\ast$ and $T_k^{**} = \Pi_k^\ast$. It follows that $W^* = W^{**}$. 

A.7 Proof of Proposition 7

By computing the first-order conditions of the maximization problem (16) and solving the resulting equations systems, we derive the access charges in the passenger segment as:

$$a_p^A = \frac{2v(1 + \lambda)(1 + c_p/2) + \theta_p(2\lambda(1 + c_p/2) - 1)}{c_p(1 + 2\lambda) + 4\lambda + 1} \quad (\text{Regime A}),$$

$$a_p^B = \frac{v + \theta_p}{2} + \frac{v - \theta_p}{2(c_p(1 + \lambda) + 4\lambda + 3)} \quad (\text{Regime B}).$$

22Remember that operator $i$ in segment $k$ realizes zero profits because $T_{ik} = (p_k - a_k)q_{ik} - 1/2c_kq_{ik}^2$. 

24
The access charges in the freight segment are given in both regimes by:

\[
a^A_f = \frac{v + \theta_f}{2} + \frac{v - \theta_f}{c_f(1 + \lambda) + 3\lambda + 1} \quad \text{(Regimes A and B)}.
\]

Let \( \varphi = (c_p(1 + 2\lambda) + 4\lambda + 1)(c_p(1 + \lambda) + 4\lambda + 3) \).

ad (i) We compute \( a^A_p - a^B_p = -\frac{\varphi}{2}(2 + c_p)^2(1 + \lambda)(\theta_p - v) < 0 \). Thus, access charges are higher in Regime B than in Regime A.

ad (ii) Note that governmental transfers are given by \( T^s = F + (v - a_p)\hat{q}_p + 2(v - a_f)\hat{q}_f \) in Regime \( s \in \{ A, B \} \). Substituting equilibrium access charges from Regimes A and B in \( T^s \), we compute \( T^A - T^B = \frac{1}{\varphi}(2 + c_p)(1 + \lambda)(\theta_p - v)^2 \left[ 5 + 8\lambda + c_p(5 + c_p + \lambda(6 + c_p)) \right] > 0 \). Thus, governmental transfers are higher in Regime A than in Regime B.

ad (iii) Substituting equilibrium access charges from Regimes A and B in the welfare function, we compute \( W^A - W^B = \frac{1}{2\varphi}(2 + c_p)^2(1 + \lambda)^2(\theta_p - v)^2 > 0 \). Thus, social welfare is higher in Regime A than in Regime B.

References


CER and EIM (2008), Rail Charging and Accounting Schemes in Europe - Case Studies from Six Countries, Technical report, Community of European Railways and European Rail Infrastructure Managers.


OECD (2005), Railway reform and charges for the use of infrastructure, Technical report, OECD.


